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Step-wise magnetic responses in mesoscopic spin-glasses

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Abstract
We study the statistics of the magnetic responses in mesoscopic spin-glass samples based on a class of mean-field models. The magnetization of a given sample grows in a step-wise manner as the external magnetic field is increased, providing a fingerprint of a given sample. We show that the statistics of sample-to-sample fluctuations of linear and nonlinear susceptibilities encode in compact ways basic scales of the steps: typical height, width and spacing. In the thermodynamic limit, the spacing between the steps vanishes, leading to chaotic changes of underlying equilibrium states by arbitrarily small but finite variations of the applied field while the magnetization per spin at a given external field converges to a self-averaging value. We also discuss possible modifications needed in realistic finite-dimensional systems guided by a Migdal–Kadanoff renormalization group analysis.

1. Introduction

Magnetization in frustrated magnets often shows interesting nonlinear responses under variation of external magnetic field, such as the devilish staircases [1]. Although such a feature is seemingly absent in spin-glasses, an early numerical study [2] of the Sherrington–Kirkpatrik (SK) model has suggested that mesoscopic spin-glass samples may exhibit characteristic step-wise responses. Measurements of conductance fluctuations in a metallic spin-glass [3] have revealed rich aspects of magnetic fluctuations at mesoscopic scales including changes of fluctuations under variation of magnetic field. On the other hand it is well known that variations of the applied field by a very small amount can strongly perturb the relaxation process manifested as the well known difference between field-cooled and zero-field-cooled magnetization [4] and rejuvenations induced by variations of magnetic field [5, 6]. Since the

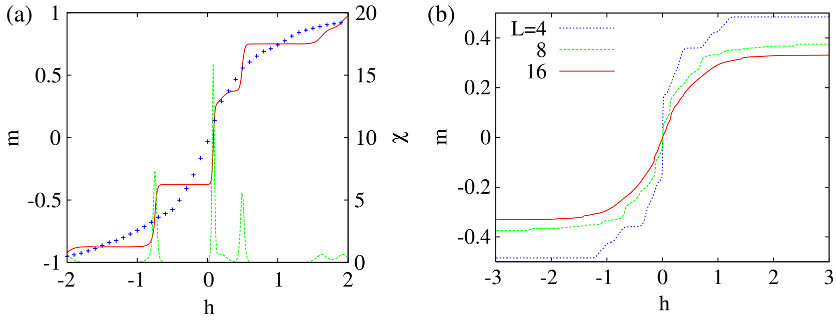

Figure 1. Magnetic responses in some samples of finite-sized spin-glass models. Magnetization per spin $m = M/N$ with $M = \sum_i \langle S_i \rangle$ of (a) $p = 3$ Ising MFSG model of $N = 16$ spins at $T = 0.1$ and (b) the Edwards–Anderson Ising spin-glass model on a hierarchical lattice of sizes $L = 4, 8, 16$ at $T/T_c = 0.1$. Data in (a) and (b) are obtained by taking traces over the Ising spin variables numerically. In (a) the spikes are the linear susceptibilities $\chi = N^{-1}(\langle M^2 \rangle - \langle M \rangle^2)/T$ and the dotted line is the disorder-average of the magnetization curve. Typical spacing between the steps is $h_s \sim T_c/\sqrt{N q_{\mathrm{EA}}}$ (see section 2). A macroscopic sample $N \gg 1$ will have a magnetization curve with infinitely many steps which is indistinguishable with the disorder-averaged one. In (b) the spacing between the steps is $h_s \sim \Upsilon/\sqrt{q_{\mathrm{EA}}} L^{-\zeta}$ (see section 3). (NB: The model in (b) has $S \leftrightarrow -S$ symmetry so that $\langle M \rangle_0 = 0$ in each sample.)

(This figure is in colour only in the electronic version)

length scales explored in these experiments are at most 10–100 lattice spacings because of the slow dynamics, such experiments are also probing mesoscopic scales. In order to understand better the magnetic responses in mesoscopic spin-glass samples, we study the problem based on a class of mean-field models (section 2) and a Migdal–Kadanoff renormalization group (MKRG) scheme (section 3).

2. Mean-field theoretical approach

We consider a class of mean-field spin-glass (MFSG) models with p-spin quenched random interactions $J_{i_1,\ldots,i_p}$ and uniform external field h,

$$H = -\sum_{i_1 < i_2 < \cdots i_p} J_{i_1,\ldots,i_p} S_{i_1} \ldots S_{i_p} - h \sum_i S_i. \tag{1}$$

with Ising spins $S_i (i = 1, \ldots, N)$. The coupling $J_{i_1,\ldots,i_p}$ follows a Gaussian distribution with zero mean and variance $J\sqrt{\frac{p!}{2N^{p-1}}}$. In the following we denote thermal averages under external field h as $\langle \cdots \rangle_h$ and disorder-averages, i.e. averages over different realizations of the spin-glass sample, as $[\cdots]_{\mathrm{av}}$. We denote inverse temperature as $\beta = 1/T$.

We consider spin-glass phases of the family of $p > 2$ models [7, 8] characterized by one-step replica symmetry breaking (1RSB). Compared with the full RSB phases (e.g. such as that of the SK model ($p = 2$)) 1RSB phases are easier to analyse.

As shown in figure 1, the magnetization exhibits step-wise responses resembling very much the one observed in the SK model [2]. A natural scenario is that it reflects level-crossings of low-lying metastable states induced by changes of the external fields. Since the susceptibilities are significant only in the close vicinities of the steps, the presence of the steps appears to be intimately related to the result of the RSB theory [9] that linear susceptibilities in mean-field spin-glass models are non-self-averaging [10]. On the other hand, in a macroscopic sample the response *must* be self-averaging. This may be realized by vanishing of the spacing between the steps as the system size is increased [2, 10]. Then it means infinitely large number of level-crossings or 'first-order phase transitions' underly an apparently smooth macroscopic response. Most probably they lead to chaotic changes of the equilibrium states. We clarify the above expectations on a firmer ground within the class of mean-field models characterized by 1RSB.

We begin by considering a generalized complexity [11], $\Sigma(f,m) = \frac{1}{N}\sum_\alpha \delta(m_\alpha - m)\delta(f_\alpha - f)$, where f_α, m_α are free-energy per spin and magnetization per spin of a metastable state (TAP state) α. As we reported recently in [11], it can be shown that under zero external field $h = 0$, close to the zero complexity plane $\Sigma(f,m) = 0$,

$$\mathrm{e}^{N\Sigma(f,m)} \propto \mathrm{e}^{xF - \frac{M^2}{2Nq_{\mathrm{EA}}}} \tag{2}$$

with $F = Nf$ and $M = Nm$. For simplicity we have chosen the minimum free-energy to be 0. q_{EA} is the Edwards–Anderson (EA) order parameter and the parameter $x \geqslant 0$ is identified to the x parameter which characterizes RSB. In the family of $p > 2$ models which exhibit 1RSB one finds $x = 1/T_{\mathrm{c}}$, with T_{c} being the critical temperature.

Let us first recall the magnetic response at macroscopic scales. Usually one computes it as follows. First one computes the thermodynamic free-energy under finite external field h in the thermodynamic limit. Then one takes derivatives of it with respect to the external field h and obtains the macroscopic magnetization which is self-averaging by definition. As we reported in [11], the generalized complexity equation (2) allows one to obtain the same macroscopic response. The crucial point is that at any value of the field h the relevant group of TAP states which dominate the equilibrium measure in the glassy phase must have zero complexity, $\Sigma = 0$. The macroscopic response is obtained by considering minimization of the total free-energy $f - hm$ on the zero complexity line $\Sigma(f,m) = 0$. Then one finds that variation of the field h induces extensive level-crossings [11]: an arbitrary small but finite variation of h select different groups of TAP states with different values of the f and m. Since they certainly have zero overlap with respect to each other, chaos underlies the smooth macroscopic response.

Now we turn to mesoscopic responses. We will not take the thermodynamic limit $N \to \infty$ from the beginning but rather keep it large but finite and consider the response to very small variation of the applied field.

First let us label the TAP states as $l = 0, 1, 2, \ldots$ such that their free-energies F_ls are ordered as $F_0 \leqslant F_1 \leqslant \cdots$. An important piece of information is the distribution of the gaps between the free-energy levels $\Delta F_l = F_l - F_{l-1} (l = 1, 2, \ldots)$. Note that equation (2) means that the free-energies follow an exponential distribution. Recently it has been proved on general grounds [12] that the gaps between such ordered random variables drawn from an exponential distribution function follow level-dependent exponential distributions. In our present problem the distribution functions reads as $\rho_l(\Delta F_l) = T_{\mathrm{c}}^{-1}\exp(-l\frac{\Delta F_l}{T_{\mathrm{c}}})$. The magnetization of the lth state, which we denote as M_l, is independent of F_l and follows a Gaussian distribution with zero mean and variance $\sqrt{Nq_{\mathrm{EA}}}$. (See equation (2).)

Let us consider the evolution of the low-lying TAP state under variation of the external field h. We assume that TAP states can be continued under variation of h, which is plausible for the 1RSB models in which TAP states relevant in equilibrium are not marginal but questionable in full RSB models since the relevant TAP states are marginal.

To obtain some basic insights we consider a two-level model which consists of only the two lowest levels $l = 0, 1$. The free-energy gap $F_1 - F_0 = \Delta F$ is only of order O(1) while the difference in their magnetizations $\Delta M = M_1 - M_0$ is of order $\sqrt{N}$.

The response $\langle M\rangle_h - \langle M\rangle_0$ can be formally expanded in a power series of h as $\langle M\rangle_h - \langle M\rangle_0 = \sum_{k=0}^{\infty}\frac{\tilde{\chi}_k}{k!}h^k$ $\chi_k \equiv \frac{\partial\langle M\rangle_h}{\partial h}|_{h=0} = \Delta M\tilde{\chi}_k(\frac{h-h_{\mathrm{s}}}{h_{\mathrm{w}}})\frac{1}{h_{\mathrm{w}}^k}$ with $\tilde{\chi}_k(y) \equiv \frac{\partial^k}{\partial y^k}\frac{1}{1+\mathrm{e}^{-y}}$. Here we have identified two characteristic scales of the external field, the 'distance to critical field', where a level-crossing and thus a step in the response takes place, $h_s \equiv \frac{\Delta F}{\Delta M} \sim O(\frac{T_c}{\sqrt{N q_{EA}}})$, and the width of thermal rounding of the step $h_w \equiv \frac{T}{\Delta M} \sim O(\frac{T}{\sqrt{N q_{EA}}})$.

From the above expansion of the response we find that the linear and nonlinear susceptibilities χ_k are significant only in the close vicinities of the step (see figure 1(a)). Another important point suggested by the expansion is that the small field expansion will converge only for small enough h/h_w. This means that the static fluctuation dissipation theorem (FDT), which assumes such an expansion, make sense only over the range of the thermal width of steps h_w which vanishes in the thermodynamic limit $N \to \infty$.

Based on the above observations, the strength of sample-to-sample fluctuations of the linear and nonlinear susceptibilities at $h = 0$ can be estimated roughly as follows. With probability $h_w/h_s \sim O(T/T_c)$ a given sample has a 'critical field' h_c within the range of the thermal width h_w around $h = 0$. With such a probability the sample has a significant susceptibility $\chi_k \sim \Delta M h_w^{-k}$. As the result we obtain a generic scaling form for the disorder-average of the lth moment of nth susceptibility as

$$[\chi_k^l]_{av} \sim (\Delta M h_w^{-k})^l \frac{h_s}{h_w} \sim \beta^{kl} \left[(N q_{EA})^{(1+k)/2}\right]^l \frac{T}{T_c}. \quad (3)$$

Thus susceptibilities are dominated by rare events which happen with small probability T/T_c and as a result they are non-self-averaging. Moreover, fluctuations of higher nonlinear susceptibilities diverge strongly with increasing N, reflecting the 'staircases'.

It can be proved [13] that the more generalized M-level model gives exact results up to $O((T/T_c)^{M-1})$ by a systematic low-temperature expansion analysis. Thus the above scaling based on the simple $M = 2$ level model shown above is actually correct up to $O(T/T_c)$, which is sufficient at low enough temperatures.

The anomalous scaling of the sample-to-sample fluctuations of the susceptibilities equation (3) can be proved directly by the replica method without invoking the TAP approach. Using the replica trick [9] we obtain an identity $\kappa_k(M) = \frac{\partial^k}{\partial(\beta\delta h)^k} \log Z(T, \delta h) = \lim_{n\to 0} \frac{\partial^k}{\partial(\beta\delta h)^k} \frac{Z^n(T,\delta h)}{n}|_{\delta h=0}$ where $Z \equiv \prod_i \{\sum_{S_i=\pm 1}\} \exp(-\beta(H - \delta h \sum_i S_i))$ is the partition function with H being the Hamiltonian without probing field δh. $\kappa_k(M)$ is the cumulant correlation function of the magnetization M. The susceptibility χ_k is related to the cumulant as $\chi_k = \beta^k \kappa_k(M)$, which is nothing but the static FDT. Then disorder-averages of the lth moment of χ_k are obtained as

$$[\chi_k^l]_{av} = \beta^{kl} \lim_{n_1,n_2,\ldots,n_l \to 0} \frac{\partial^k}{\partial(\beta\delta h_1)^k} \frac{\partial^k}{\partial(\beta\delta h_2)^k} \cdots \frac{\partial^k}{\partial(\beta\delta h_l)^k} \left. \frac{[Z_1^{n_1} Z_2^{n_2} \ldots Z_l^{n_l}]_{av}}{n_1 n_2 \cdots n_l} \right|_{\delta h=0}. \quad (4)$$

We first evaluate the rhs of the above equation regarding $n_1, n_2, \ldots, n_l$ as integers which amounts to introducing l groups of replicas $r = 1, 2, \ldots, l$ which consist of $n_1, n_2, \ldots, n_l$ replicas. We label them by two indices such as $(r, 1), (r, 2), \ldots, (r, n_r)$. Within the class of models which exhibit one RSB one can show in general [13] that,

$$[Z_1^{n_1} Z_2^{n_2} \ldots Z_l^{n_l}]_{av} = \text{const} \sum_{SP} \left[\frac{N(\Delta_{eff}^2)}{2} \sum_{r,s=1}^{l} \beta\delta h_r \beta\delta h_s \sum_{\alpha=(r,1)}^{(r,n_r)} \sum_{\beta=(s,1)}^{(s,n_s)} \delta_{i_\alpha, i_\beta} \right] \quad (5)$$

where we introduced index i as the label of clusters in a given one RSB saddle point (SP). The sum $\sum_{SP}$ stands for summation over SP solutions obtained by permutations of the Parisi's matrix. In the following we only consider the susceptibilities at $h = 0$ where the parameter Δ_{eff} is identical to the Edwards–Anderson order parameter q_{EA}. The size of each cluster in the one RSB ansatz is given by $xT = T/T_c$.

For the linear susceptibility we obtain the disorder-average as $[T\chi_1]_{av} = N q_{EA} x T$ and the disorder-average of the second moment $[(T\chi_1)^2]_{av} = (N q_{EA})^2 x T$. The results agree with the scaling equation (3). In particular, we find $\sqrt{[\chi_1^2]_{\rm av} - [\chi_1]_{\rm av}^2} / [\chi_1]_{\rm av} = 1 - xT$, so that the linear susceptibility is not self-averaging below $T_{\rm c}$ as found in the SK model [10].

Furthermore, for the nonlinear susceptibilities we find stronger sample-to-sample fluctuations in agreement with equation (3). The average of the first nonlinear susceptibility vanishes ($[T^2\chi_2]_{\rm av} = 0$) for simple symmetry reasons. However, for the second moment we obtained after tedious but straightforward computations $[(T^2\chi_2)^2]_{\rm av} = (Nq_{\rm EA})^3 2xT(1-xT)$, in agreement with the scaling equation (3).

Note that the results are obtained as polynomials of $xT = T/T_{\rm c}$ so that the expressions can be regarded as low-temperature expansions. Indeed we performed the low-temperature expansion analysis mentioned previously up to $O((T/T_{\rm c})^2)$ and found precise agreements with the above results up to that order [13].

3. Migdal–Kadanoff real space renormalization group approach

Let us now turn to an analysis of magnetization curves in an EA Ising spin-glass model defined on a hierarchical lattice designed to mimic a $d = 3$ dimensional regular lattice of size L with $N = L^{d=3}$ spins. The Hamiltonian of the model is the same as equation (1) but with $p = 2$ body spin–spin couplings given only on nearest-neighbour pairs. A real space renormalization group procedure works exactly on the hierarchical lattice including also renormalization of the magnetic field [14]. As shown in figure 1(b), the magnetization grows with increasing field in a step-wise manner. The number of steps increases and the magnetization curve converges to a self-averaging one as the system size N is increased. Interestingly these features are essentially the same as in the mean-field case.

Variation of the field changes renormalized couplings and fields and thus induces level-crossings of low-lying states leading to step-wise responses. Low-lying states which are related to each other by cluster flips or droplet excitations [15] of a given length scale L have differences in the magnetizations of order $\sqrt{N\Delta_{\rm eff}}$ with $N = L^d$. A subtle point is that the typical energy gap between low-lying states is not $T_{\rm c}$ but now it is ΥL^{θ} with stiffness constant $\Upsilon > 0$ and stiffness exponent $\theta > 0$. In the present $d = 3$ case $\theta \simeq 0.26$. Thus one finds again $h_{\rm w} \sim T/\sqrt{N\Delta_{\rm eff}}$ but $h_{\rm s} \sim \Upsilon/\sqrt{\Delta_{\rm eff}} L^{-\zeta}$ with $\zeta = d/2 - \theta$.

Let us list below other important differences compared with the mean field case. (1) Low-lying excitations exist not only on the scale L of the sample itself but on smaller scales $L/2, L/4, \ldots$. Since the linear susceptibility is dominated by contributions from short length scales it becomes self-averaging while higher-order nonlinear susceptibilities can still be non-self-averaging. (2) At strong fields $|h| \gg h_s$, actually renormalized couplings at the scale L of the samples vanish so that the samples behave as paramagnets [14] in agreement with the prediction of the droplet-scaling theory [15]. For the length scales $L = 4$–16 of the sample used in figure 1(b), $h_{\rm s} = 0.18$–0.03. It is interesting to note that step-wise responses continue up to stronger fields. This is because renormalized fields acting on the scale L fluctuate on variation of the applied field h due to level-crossings at smaller length scales where $h_{\rm s}$ is larger.

4. Conclusion

To summarize, we analysed a class of mean-field models which exhibit 1RSB and found analytically that the statistics of sample-to-sample fluctuations of linear and nonlinear susceptibilities are indeed intimately related to the anticipated step-wise responses. The basic scales of the steps (typical height $\sqrt{N\Delta_{\rm eff}}$, width $h_{\rm w}$ and spacing $h_{\rm s}$) are encoded in compact ways in a scaling form. We compared the results with an MKRG analysis.

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